

Integrated optimization model of production planning and scheduling for batch production

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ABSTRACT

An integrated optimization model of production plan and scheduling is introduced. Function objective is to minimize sum of total setup cost (initial setup cost and related setup cost), stock cost, production cost and overtime cost. Restraint functions include equilibrium stock, production capacity, as well as scheduling restraints (procedure restriction and machine capacity restriction). Feasible plan can be set down with considering scheduling restraints.

Key words: integrated optimization, production plan, scheduling, schedule restraints

1. INSTRUCTION

In machine tool Industry, machine shop can be differentiated into two types: small part with large batch and big part with small batch. The former shop floor produces machine tool axes, cover, tray and special figure part, type of product vary a lot, lot size is large, production cycle is short, plan change frequently, lot size includes monthly batch, semi-monthly batch, weekly batch and daily batch as a result of large lot size delivered and frequently stock. Enterprise plan includes yearly production plan, quarterly production plan and monthly production plan. Manufacturing department in factory deliver monthly product plan to shop floor, and monthly product plan is carried into execution in shop floor. Planner in shop floor set down shorter part production planning, they generally confirm part lot size and turn of delivering parts by experience, thereby result in large work-in-process, infeasible plan and imbalance production. Traditional hierarchical plan decomposed in MRPII often results in infeasible plan in scheduling system as for without considering schedule restraints, new method to establish plan must be adopted.

The kernel goal of plan in batch shop floor is to ascertain part lot size during each period and minimize sum of total setup cost and stock cost. For batch production, literature[1~4] proposed integrated optimization method for production planning and scheduling. Literature [1] put forward integrated model of production planning and scheduling in shop floor, decompose planning alternatively for instances with setup time and without setup time in Job-shop, but the method can only convergence at local optimization results. Literature [2] set up an integrated model of production planning and scheduling in shop floor, schedule restrictions as procedure restraint and machine capacity restraint are taken into account, and its Lagrange relaxation-based solution approach is described, but the method is hard to solve large scale problem for the complexity of the arithmetic to relax schedule sub-problem. Literature [3] give a heuristic

method aiming at complex assembly product manufacturing, optimization lot size is made certain by grouping orders and schedule by searching primary meshwork routes, and function objective is to minimize sum of setup time, stock cycle and total delivery cycle, but lot size in multi-periods is not discussed. In literature [4], an integrated general model of lot-sizing and scheduling is described for multi-stage, multi-resources flexible flow-line, function objective is to minimize sum of total setup cost, stock cost and overtime cost, but production cost and schedule restraints are not considered.

According as merit and demerit of above-mentioned integrated model for production planning and scheduling in batch manufacturing, an integrated optimization model of production planning and scheduling for batch production is recommended, function objective is to minimize sum of total setup time, stock cost, production cost and overtime cost, schedule restrictions include stock equilibrium, procedure restraints, production capacity and machine capacity. The model is non-linear, mixed, integer programming. Genetic arithmetic is an overall optimization searching method with strong adaptability and better extended capacity, its convergence can be amended by linking with heuristic and other searching method, and it can be used to solve complex non-linear problem^[5]. A mixed, heuristic genetic arithmetic will be used to search results of above-mentioned model in future research.

2. INTEGRATED OPTIMIZATION OF PRODUCTION PLANNING AND SCHEDULING FOR BATCH PRODUCTION

In machining workshop of batch production in machine tool industry, type of product varies from several to many, parts processed monthly change constantly with much kinds. Machining workshop has multiplicate equipments, and per equipment has one or more machines. For variable market, shop floor production planning must be dynamic, real-time and agile. In this paper, an integrated optimization model of

production planning and scheduling is described in the context of axes workshop section in shenyang primo machine tool company, which belongs to medium batch production with much kinds. It decomposed workshop sections' monthly part production planning into weekly cell part production planning, and each part's weekly lot size is fixed on.

Axes workshop section belongs to small part with large batch production, includes four production lines: shaft, principal axis, spindle and caudal core, parts are processed separately in each production line, several machine tools are in overtime. Working procedure length, part complexity, cell equipments' capacity, production balance and assemblage of whole product are main factors considered while to set down planning. Scheduling factors include rough material, tools, equipments and staff.

Cell weekly production planning is lot-sizing problem. Lot-sizing planning is to allot batch and process time of process items collection, so as to meet definite capability target. Considering within finite plan span in shop floor, N kinds of parts are processed on M machine tools, and each part is processed on machine tool in same order. Setup time of part j on machine k is correlated to type of former part i processed. Plan span is separated into T period, and exterior requirement d_{jt} for each part in period t is known. Objective of lot-sizing planning is to arrange item lot size in each period, and to minimize sum of total setup cost, stock cost and production cost during whole plan period on the condition of meeting restrictions.

Supposing: 1) Lot size must meet exterior requirements during each period; 2) During each period, a part must be processed without interrupted, other part must be machined after the part; 3) Overtime is permitted, but excess cost brought must be considered.

Integrated optimization model of production planning and scheduling meeting above conditions is as follows:

$$J = \min \sum_{j=1}^N \sum_{k=1}^M \sum_{t=1}^T sc_{0jk} Z_{jt} + \sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^M \sum_{t=1}^T sc_{ijk} Y_{ijt} + \sum_{j=1}^N \sum_{t=1}^T (p_{jt} X_{jt} + h_{jt} I_{jt}) + \sum_{k=1}^M \sum_{t=1}^T oc_k O_{kt} \quad (1)$$

Subject to:

$$I_{j,t-1} + X_{jt} - d_{jt} = I_{jt}, \quad \forall j \in N, \forall t \in T \quad (2)$$

$$\sum_{j=1}^N \sum_{k=1}^M p_{jk} X_{jt} + \sum_{j=1}^N \sum_{k=1}^M st_{0jk} Z_{jt} + \sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^M st_{ijk} Y_{ijt} \quad (3)$$

$$\leq C_{kt} + O_{kt}, \quad \forall t \in T$$

$$Y_{ijt} B - X_{jt} \geq 0, \quad \forall i \in \{0, N\}, \forall j \in N, \forall t \in T \quad (4)$$

$$\sum_{j=1}^N Z_{jt} = 1, \quad \forall t \in T \quad (5)$$

$$X_{jt}, I_{jt}, O_{kt} \geq 0, \quad \forall j \in N, \forall t \in T, \forall k \in M \quad (6)$$

$$Y_{ijt}, Z_{jt} \in \{0, 1\}, \quad \forall i, j \in N, \forall t \in T \quad (7)$$

Known parameters : d_{jt} - independent requirements of item i in period t ; p_{jt} - unit production cost of item j in period t ; p_{jk} - machining time of item j on machine k ; st_{ijk} - setup time from item i to item j on machine k , if $i = 0$, denote initial setup time of item j on machine k ; sc_{ijk} - setup cost from item i to item j on machine k , if $i = 0$, denote initial setup cost of item j on machine k ; h_{jt} - unit stock cost of item j during period j to $j+1$; C_{kt} - usable throughput of machine k ; oc_k - overtime cost of machine k during unit time; B - a very large plus.

Decision-making variables: I_{jt} - stocks of item j on period t ; O_{kt} - overtime of machine k on period t ; X_{jt} - production quantity of item j on period t ; Z_{jt} - setup variables, if $Z_{jt} = 1$, denote setup course to process item j at the end of period t or at the beginning of period $t+1$, other instance $Z_{jt} = 0$; Y_{ijt} - setup variables, if $Y_{ijt} = 1$, $i \neq j$, denote setup course to process item j after item i processed, other instance $Y_{ijt} = 0$.

Objective function (1) is to minimize sum of total setup cost (initial setup cost + related setup cost), stock cost and overtime cost during whole plan period. Restriction (2) is essential stock balance equation. Restriction (3) is inequality of capacity restraints. Restriction (4) denotes setup can only happen while production quantity is plus and border upon items are different. Restriction (5) denotes that each item has only one initial setup. Restriction (6) denotes that decision-making variables are not minus. Restriction (7) means that setup variables are 0 or 1. But above-mentioned model not includes following schedule restrictions:

a. Procedure restraints: each item has several working procedure with definite order.

b. Capacity restraints: a machine can only process one item on any time.

To describe these schedule restraints, such parameters are defined: J_{jt} - an item batch task (j, t), namely item j processed on period t , its production quantity is X_{jt} , $\forall j \in N, \forall t \in T$; J_t - N task collection must be completed during period t , $J_t = \{J_{1t}, J_{2t}, \dots, J_{Nt}, \forall t \in T\}$; J - all the task

3. CONCLUSIONS

Motivated by the aim of improving the feasible plan level of short plan period in a real case, an integrated optimization model of production planning and scheduling has been proposed in weekly item batch confirming.

The function objective is to minimize sum of total setup cost (initial setup cost + related setup cost), stock cost and overtime cost of item within whole planning period. To ascertain lot size, detailed schedule restraints are discussed, so as to get hold of a feasible production planning.

As for schedule restrictions, procedure restraints and capacity restraints are debated. Procedure restraints are described as each item has several working procedure with definite order, and capacity restraints are put forward by way of a machine can only process an item on any time. A mixed, heuristic genetic arithmetic will be used to search the results of above-mentioned model in next research work.

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collection , $J = \bigcup_{t=1}^T J_t$; s_{jt} - setup variables,

$$s_{jt} = \begin{cases} 1, & X_{jt} > 0 \\ 0, & X_{jt} = 0 \end{cases} ; o_{jkt} - \text{operation of job } J_{jt} \text{ on}$$

machine k ; t_{jkt} - time begin to operate o_{jkt} ; OM_k - collection of all the operations on machine k , $OM_k = \{o_{jmt} | m = k, j \in N, t \in T\}$; OP_j - collection of all the operations of item j ; $OP_j = \{o_{lkt} | l = j, k \in M, t \in T\}$; p_{jkt}^u - process time of operation o_{jkt} of unit item j ; τ_{jkt} - setup time of operation o_{jkt} of unit item j ; GN - collection of all the operations with supposed incept and accomplish operations ; GA - versus collection of working procedure composed of all the operations with orders ; GE_k - versus collection of working procedure composed of all the operations on machine k .

In this way, above schedule restraints can be described as follow:

$$t_{jkt} - t_{jk't} - p_{jkt}^u X_{jt} - \tau_{jkt} s_{jt} \geq 0, \quad (8)$$

$$\forall (o_{jk't}, o_{jkt}) \in GA$$

$$t_{jkt} - t_{j'kt} - p_{jkt}^u X_{jt} - \tau_{jkt} s_{jt} \geq 0, \quad (9)$$

$$\forall (o_{j'kt}, o_{jkt}) \in GE_k$$

$$t_{jkt} \geq 0, o_{jkt} \in GN \quad (10)$$

$$s_{jt} \in \{0, 1\}, \forall j \in N, \forall t \in T \quad (11)$$

Restraint (8) denotes turns between working procedures of the same item; restraint (9) denotes turns between items processed on the same machine tool; restraint (10) shows decision-making variables are not minus; restraint (11) expresses that setup variables are 0 or 1.

Integrated optimization model of production planning and scheduling with considering lot-sizing and scheduling restraints as follow:

$$J = \min \sum_{j=1}^N \sum_{k=1}^M \sum_{t=1}^T sc_{0jk} Z_{jt} + \sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^M \sum_{t=1}^T sc_{ijk} Y_{ijt} \quad (1)$$

$$+ \sum_{j=1}^N \sum_{t=1}^T (p_{jt} X_{jt} + h_{jt} I_{jt}) + \sum_{k=1}^M \sum_{t=1}^T oc_k O_{kt}$$

Subject to: formula (2) ~ (11).