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ADAPTIVE OUTPUT FEEDBACK CONTROL FOR UNCERTAIN SYSTEMS*

XIAO-LI ZHANG, YU-SHUN FAN

Department of Automation, TsingHua University, Beijing, 100084, China

E-MAIL: zxl@cims.tsinghua.edu.cn

Abstract

The problem of adaptive output feedback stabilizing a class of systems with uncertainties and disturbances is considered. The conditions of output feedback stabilizing system are given. Uncertainties of the system can be nonlinear or time-varying. They satisfy the so-called matching condition. The bounds of the uncertainties exist, but are unknown. A certain nonlinear controller which guarantee the states of the system to asymptotically converge to equilibrium point is designed. Finally, the simulation shows the validity of the result.

Keywords

Adaptive control; Unknown bounds of uncertainties; Output feedback; Asymptotically stable

1 Introduction

Robust stabilization of uncertain system is an important problem in control system. Usually, the stabilization of the system includes stabilization of state feedback[1]-[2] and stabilization of output feedback[3]-[4]. The former has gotten a lot of results, but it requires the complete information of the system. The states of the system can only be partly known in usually engineering practice. Output feedback is easy to perform, because the output is measurable. So it is significant to investigate the output feedback in theory and practice.

In recent years, some results have been received about stabilization of output feedback. In [4], Kwan considered a matched uncertain variable structure systems in which the state was unavailable and no estimated state was required. Shyu extended the result to the system with unmatched uncertainty[5], Choi studied a system with uncertainty in system by using variable structure output feedback control[6], but the systems in their papers have not additional disturbances. In this paper, we study the problem of adaptive output feedback stabilizing a class of systems with uncertainties and disturbances. The conditions of output feedback stabilizing system are given. The uncertainties of the system can be nonlinear or time-varying. They satisfy the so-called matching condition. The bounds of the uncertainties exist, but are unknown. A certain nonlinear controller which guarantee the states of the system to asymptotically converge to equilibrium point and adaptive variables remain bounded is designed. Finally, the simulation shows the validity of the result.

2 Description of problem and main results

2.1 Description of problem and preliminary

Consider the following uncertain systems S, we can formulate systems S as:

$$\dot{x} = Ax + B[I + E(t,x,\sigma)]u + B\Delta f(x) + Bd(t) \quad (1a)$$

$$y = Cx \quad (1b)$$

where $x(t) \in R^n, u(t) \in R^m$, $y(t) \in R^m (n \geq m)$, $d(t) \in R^m$, are the states, control input, measurable output and additional disturbance of the systems respectively. $A \in R^{n\times n}, B \in R^{n\times m}, C \in R^{m\times n}$ are state matrix, input gain matrix and output matrix of the nominal systems respectively. The uncertain parameter $\sigma \in \Omega \subset R^q$ is ***Lebesgue*** measurable. Ω is compact subset in R^q. $E(t,x,\sigma): R \times R^n \times \Omega \to R^{m\times m}$ is the uncertainty of input gain.

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$\Delta f(x)$ is nonlinear uncertainty. $I \in R^{m\times m}$ is unit matrix.

The uncertainty of system's input gain $E(t,x,\sigma)$ is piecewise continuous for variable t, and is continuous for x,σ. The uncertainty $\Delta f(x)$ is smooth for x. The additional disturbance $d(t)$ is piecewise continuous vector function. Under the above condition, we know that the systems (1) have unique solution for any initial state and piecewise continuous control input $u(t)$.

We give the following assumptions for systems (1).

Assumption 1[3] $\det(CB) \neq 0$ (nonsingular high frequency gain)

Assumption 2[3] (A,B,C) is stabilizable and detectable.

Assumption 3[3] Transfer function matrix $C(sI-A)^{-1}B$ is minimum phase.

Assumption 4 There exist nonnegative functions $\eta(t),\rho(t),\lambda(t)$ and nonnegative constant η^*,ρ^*,λ^* such that

$$\|E(t,x,\sigma)\| \le \eta(t) < 1, \tag{2a}$$

$$\|\Delta f(x)\| \le \rho(t)\|x\|, \tag{2b}$$

$$\|d(t)\| \le \lambda(t) \tag{2c}$$

where $\eta(t),\rho(t),\lambda(t)$, η^*,ρ^*,λ^* are unknown, and satisfy $\eta(t)\le\eta^*$, $\rho(t)\le\rho^*,\lambda\le\lambda^*$.

The condition $\eta^*<1$ in assumption 4 denotes that the uncertain gain in every input passage is less than normal input gain. It is reasonable.

In order to obtain the main result, the following lemma is needed.

Lemma 1 Consider systems

$$\begin{cases}\dot z = Az + Bu\\ y = Cz\end{cases} \tag{3}$$

where $z\in R^n, u, y \in R^m$; (A,B,C) is stabilizable and detectable. Systems (3) satisfy the following conditions

1) $C(sI-A)^{-1}B$ is minimum phase,

2) $\det(CB)\neq 0$.

then there exist positive definite matrixes P and nonsingular matrixes K, such that

$$(A+\alpha I-\frac{1}{2}\beta BKC)^{\mathrm T}P + P(A+\alpha I-\frac{1}{2}\beta BKC)+\gamma I<0 \tag{4}$$

$$B^{\mathrm T}P = KC \tag{5}$$

where α,β,γ are constants and the closed loop systems composed of systems (3) and controller $u=-\frac{1}{2}\beta Ky$ are asymptotically stable.

Lemma 1 is the direct corollary of theorem 2.11 and theorem 3.3 in paper [3].

2.2 Design of asymptotically stable adaptive controller and main results

We construct the following controllers

$$u = u_1 + u_2 \tag{6}$$

where

$$u_1 = -w_1(t)Ky, \tag{7a}$$

$$u_2 = \begin{cases} -w_2(t)\dfrac{Ky}{\|Ky\|}, & \|Ky\|\neq 0\\ 0, & \|Ky\| = 0\end{cases} \tag{7b}$$

control gain $w_1(t)$ is the estimate of parameter $w_1^* = \dfrac{1}{2(1-\eta^*)}[\beta+\dfrac{1}{\gamma}(\rho^*)]$, $w_2(t)$ is the estimate of parameter $w_2^* = \dfrac{\lambda^*}{1-\eta^*}$, K is given from (5). Adaptive variables, which are adopted, are described by the following equations

$$\dot w_1(t) = \delta_1\|Ky\|^2, \qquad w_1(0)\,\hat=\, w_{10}\ge 0 \tag{8a}$$

$$\dot w_2(t) = \delta_2\|Ky\|, \qquad w_2(0)\,\hat=\, w_{20}\ge 0 \tag{8b}$$

where δ_1,δ_2 are positive constant, w_{10}, w_{20} are nonnegative limited constant.

If systems (1) satisfy assumptions 1-4, the equilibrium point of systems (1) is $x=0$ and the equilibrium point of closed loop systems consisted of systems (1), controllers (6) and adaptive controlling rules (8) is $(0,w_1,w_2)$, w_i ($i=1,2$) is any real. So, we want to prove asymptotical stability of systems (1) if and only if we prove asymptotical stability of closed loop systems which consist of systems (1) and adaptive controlling rules (8), and w_i ($i=1,2$) are uniformly bounded.

Therefore, we can get the following theorem.

Theorem 1 If uncertain systems (1) satisfy assumptions 1-4, the states of the systems (1) will asymptotically converge to zero. Adaptive variable w_1, w_2 remain bounded under the conditions of output feedback controllers (6) and adaptive controlling rules (8). The change rate of adaptive variable converge to zero when time t trend to infinite. i.e. $\lim\limits_{t\to+\infty}\dot w_i(t)=0$. ($i=1,2$)

Proof: We take controllers (6) into systems (1)

$$\dot x = Ax + B[I+E(t,x,\sigma)](-w_1(t)Ky - w_2(t)\frac{Ky}{\|Ky\|}) + B\Delta f(x) + Bd(t) \tag{9}$$

In order to study the stability of closed loop systems composed of systems (1), adaptive controlling rules (8) and controllers (6), we construct the following ***Lyapunov*** function

$$V(x)=x^{\mathrm{T}}Px+(1-\eta^*)\delta_1^{-1}(w_1-w_1^*)^2+(1-\eta^*)\delta_2^{-1}(w_2-w_2^*)^2 \quad (10)$$

For convenience of proof, let $V(t)=V_1(t)+V_2(t)+V_3(t)$, where

$$V_1(x)=x^{\mathrm{T}}Px\,, \quad (11a)$$

$$V_2(t)=(1-\eta^*)\delta_1^{-1}(w_1-w_1^*)^2\,, \quad (11b)$$

$$V_3(t)=(1-\eta^*)\delta_2^{-1}(w_2-w_2^*)^2\,, \quad (11c)$$

then the derivative of ***Lyapunov*** function $V(t)$ along systems (9) is

$$\dot V(t)=\dot V_1(t)+\dot V_2(t)+\dot V_3(t) \quad (12)$$

From assumption 2 and lemma 1, we can get

$$\dot V_1=\dot x^{\mathrm{T}}Px+x^{\mathrm{T}}P\dot x$$
$$=(2x^{\mathrm{T}}PAx+2x^{\mathrm{T}}PB(I+E(t))(-w_1Ky-w_2\frac{Ky}{\|Ky\|}) + 2x^{\mathrm{T}}PB\Delta f(x)+2x^{\mathrm{T}}PBd(x)) \quad (13)$$

$$\le -2\alpha x^{\mathrm{T}}Px-\gamma\|x\|^2+\beta x^{\mathrm{T}}PBB^TPx+2x^{\mathrm{T}}PB\times [-w_1(t)Ky-w_2(t)\frac{Ky}{\|Ky\|}]+E(t)(-w_1(t)Ky-w_2(t)\times \frac{Ky}{\|Ky\|})+\Delta f(x)+d(t)]\,, \quad (14)$$

From assumption 4, lemma 1 and pay attention to the following fact

$$2ab\le ca^2+\frac{1}{c}b^2 \quad (15)$$

the above inequality holds for any real a,b and positive real c, then we have

$$2x^{\mathrm{T}}PB(-w_1(t)Ky-w_2(t)\frac{Ky}{\|Ky\|})\le -2w_1(t)\|B^{\mathrm{T}}Px\|^2 - 2w_2(t)\|B^{\mathrm{T}}Px\| \quad (16a)$$

$$2x^{\mathrm{T}}PBE(t)(-w_1(t)Ky-w_2(t)\frac{Ky}{\|Ky\|})\le 2\eta^*(w_1(t)\|B^{\mathrm{T}}Px\|^2+w_2(t)\|B^{\mathrm{T}}Px\|) \quad (16b)$$

$$2x^{\mathrm{T}}PB\Delta f(x)\le 2x^{\mathrm{T}}PB\rho(t)\|x\|\le\frac{1}{\gamma}(\rho^*)^2\|B^{\mathrm{T}}Px\|^2+\gamma\|x\|^2 \quad (16c)$$

$$2x^{\mathrm{T}}PBd(t)\le 2\lambda^*\|B^{\mathrm{T}}Px\| \quad (16d)$$

Then we have inequality for $\dot V_1$

$$\dot V_1\le -2\alpha x^{\mathrm{T}}Px-2(1-\eta^*)w_1(t)\|B^{\mathrm{T}}Px\|^2 - 2(1-\eta^*)w_2(t)\|B^{\mathrm{T}}Px\|+\beta\|B^{\mathrm{T}}Px\|^2+\frac{1}{\gamma}(\rho^*)^2\times \|B^{\mathrm{T}}Px\|^2+2\lambda^*\|B^{\mathrm{T}}Px\|$$
$$\le -2\alpha x^{\mathrm{T}}Px+[-2(1-\eta^*)w_1(t)+\beta+\frac{1}{\gamma}(\rho^*)^2]\|B^{\mathrm{T}}Px\|^2+ [-2(1-\eta^*)w_2(t)+2\lambda^*]\|B^{\mathrm{T}}Px\| \quad (17)$$

From (11b), we have

$$\dot V_2(t)=2(1-\eta^*)\delta_1^{-1}(w_1-w_1^*)\dot w_1 = 2(1-\eta^*)(w_1-w_1^*)\|B^{\mathrm{T}}Px\|^2 \quad (18)$$

From (11c), we have

$$\dot V_3(t)=2(1-\eta^*)\delta_2^{-1}(w_2-w_2^*)\dot w_2 = 2(1-\eta^*)(w_2-w_2^*)\|B^{\mathrm{T}}Px\| \quad (19)$$

From (17)-(19), and take note of w_1^*, w_2^*, then

$$\dot V(t)\le -2\alpha x^{\mathrm{T}}Px\le 0 \quad (20)$$

equal-sign holds if and only if $x=0$, i.e. $\dot V(t)$ is negative definite.

From (13), we can get

$$0\le V(x(t),w_1(t),w_2(t))\le V(x(0),w_1(0),w_2(0)) = x^{\mathrm{T}}(0)Px(0)+(1-\eta^*)\delta_1^{-1}(w_1(0)-w_1^*(0))^2+ (1-\eta^*)\delta_2^{-1}(w_2(0)-w_2^*(0))^2=V(0)<+\infty \quad (21)$$

$V(t)$ is bounded. From above form, we know

$$x^{\mathrm{T}}Px+(1-\eta^*)\delta_1^{-1}(w_1-w_1^*)^2+(1-\eta^*)\times \delta_2^{-1}(w_2-w_2^*)^2\le V(0)\,. \quad (22)$$

Denote $\mu=\lambda_{\min}(P),\nu=\lambda_{\max}(P)$ as the minimum and maximum eigenvalue, then there is inequality for positive definite matrix P, such that

$$\mu\|x\|^2\le x^{\mathrm{T}}Px\le\nu\|x\|^2 \quad (23)$$

where μ,ν are positive real. From (22)-(23), we have

$$\|x(t)\|\le\sqrt{\frac{V(0)}{\mu}} \quad (24a)$$

$$\|w_i(t)-w_i^*(t)\|\le\sqrt{\frac{\delta_iV(0)}{1-\eta^*}} \quad (24b)$$

From (24), we can know state $x(t)$ and adaptive variable $w_i(t)$ ($i=1,2$) are ultimately uniformly bounded under the controller (6). Then

$$\int_0^t 2\mu\alpha\|x(\tau)\|^2 d\tau \le V(0)-V(t) \le V(0) \quad (25)$$

and $\int_0^t 2\mu\alpha\|x(\tau)\|^2 d\tau$ is monotonously decreasing and bounded for variable t. Then $\int_t^{+\infty} 2\mu\alpha\|x(\tau)\|^2 d\tau$ exists and $\|x(t)\|$ is bounded. From (9), (25) and assumption 4, we know that $\|\dot{x}(t)\|$ is bounded. So $\frac{d(\|x(t)\|^2)}{dt}$ is bounded, then $\|x(t)\|^2$ is uniformly continuous for time t. Therefore $\lim_{t\to+\infty}\|x(t)\| = 0$. i.e. systems (1) are asymptotically stable, and $\lim_{t\to+\infty} \dot{w}_i(t) = 0$ from equality (8).

3 Simulation

Consider the following linear dynamic systems with additional disturbances

$$\dot{x} = Ax + B(I+E)u + B\Delta f(x)(x) + Bd(t)$$
$$y = Cx \quad (26)$$

Take $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $E = -0.4\sin(10t)$, $\Delta f(x) = (\sin(5t) \quad \cos(5t))x = x_1\sin(5t) + x_2\cos(5t)$, $d(t) = 0.5\cos t$, $C = (1 \quad 1)$.

Assume $\alpha = 0.2$, $\beta = 3.4$, $\gamma = 0.5$, take $P = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$, $K = 1$, then the lemma 1 is satisfied. Let initial conditions are $x(0) = (1 \quad -1)^{\mathrm{T}}$, $w_1(0) = w_2(0) = 0$. Turning parameters are $\delta_1 = 0.35$, $\delta_2 = 0.25$. Figures 1-2 show the results of simulation

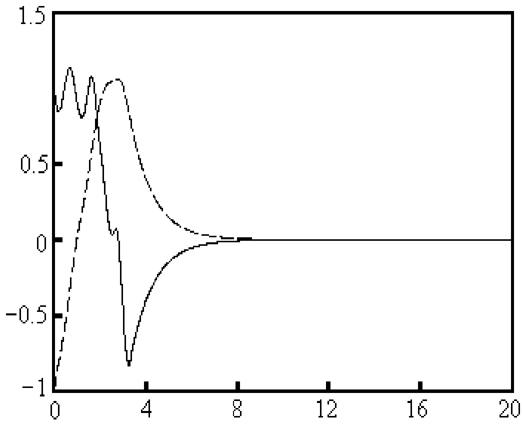

Figure 1: States of systems "x_1 __", x_2 _·_"

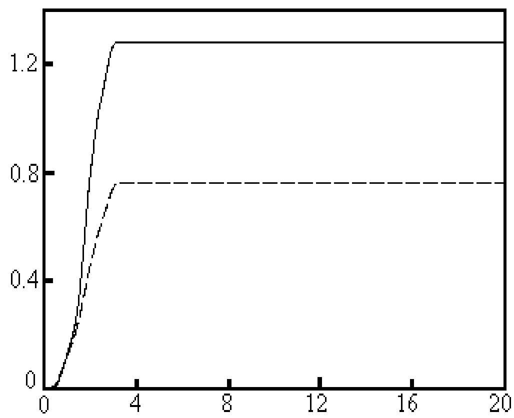

Figure 2: Adaptive variable "w_1 __, w_2 _·_"

From the figures, we can see states of the systems getting a fluctuate in the first 3 seconds, because there are additional disturbances. The controllers can not get information of output promptly. When the controllers get the information of output, the states of system decline to zero immediately under the effect of controllers.

4 Conclusion

A class of uncertain system with additional disturbance is considered in this paper. Designing methods of nonlinear and adaptive controllers are given. The designing controllers have stronger robustness from the result of this paper, and they avoid the disadvantages of the conservative and can not stabilizing the system of former controllers which are designed by the estimate bound of uncertainty. The simulations show the validity of our designing methods in this paper.

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