Proceedings of the Second International Conference on Machine Learning and Cybernetics, Xi'an, Nov.2003, pp.980-984

ADAPTIVE STATE FEEDBACK CONTROL OF EXPONENTIALLY PRACTICAL STABILITY FOR UNCERTAIN SYSTEMS*

XIAO-LI ZHANG, YU-SHUN FAN

Department of Automation, TsingHua University, Beijing, 100084, China

E-MAIL: zxl@cims.tsinghua.edu.cn

Abstract

The problem of adaptive state feedback exponentially practically stabilizing a class of time-varying systems with uncertainty and disturbance is considered. The conditions of state feedback exponentially practically stabilizing systems are given. The uncertainties of the systems satisfy the so-called matching condition. They can be nonlinear or time-varying. The bounds of the uncertainty exist, but are unknown. A kind of certain nonlinear controller which can guarantee that the states of the systems exponentially converge to the equilibrium's nearby region which was named in advance in limited time is designed. The simulation shows the validity of the result.

Keywords

Adaptive control; Unknown bounds of uncertainties; State feedback; Practically stable

1 Introduction

Robust stabilization of the uncertain systems is an important problem in control systems. State feedback is one of the methods which are often used in stabilizing systems. In recent years, many results which relate to the state feedback stabilization have been obtained. Several state feedback controllers are designed by Lyapunov method in [1-3], but the bounds of the uncertainties are known and the designing controllers are based on those bounds. However, it is very difficult to estimate the bounds of uncertainty for the real systems. So this kind of method has more limitation. The design of a kind of saturated adaptive controller is given by adaptive control strategy in [4] for the case that the bounds of the uncertainties are partially known. By using adaptive method, controller of exponentially practical stability is designed in [5]. This design of controller is invalidation in some cases because the design depends on the bound of the uncertainties of input gain. In this paper, the problem of adaptive state feedback exponentially practically stabilizing a class of time-varying systems with uncertainty and disturbance is considered. The conditions of state feedback exponentially practically stabilizing systems are given. The uncertainties of the systems satisfy the so-called matching condition. They can be nonlinear or time-varying. The bounds of the uncertainty exist, but are unknown. A kind of certain nonlinear controller which can guarantee that the states of the systems exponentially converge to the equilibrium's nearby region which was named in advance in limited time is designed. The controller avoids the disadvantage that the asymptotically stable controller can not guarantee the states of the systems converge to equilibrium point in limited time. Finally, the simulation shows the validity of the result.

2 System description and main result

2.1 System description and preliminary

We consider the following uncertain systems S, we can formulate systems S as:

$$\dot{x}(t) = A(t)x(t) + B(t)[I + E(t,x,\sigma)]u(t) + B(t)\Delta f(x) + B(t)d(t) \qquad (1)$$

where $x(t) \in R^n, u(t) \in R^m (n \geq m)$, $d(t) \in R^m$, are the states, control input and additional disturbance of the systems respectively. $A(t) \in R^{n\times n}, B(t) \in R^{n\times m}$ are state matrix and input gain matrix of the nominal systems respectively. The uncertain parameters $\sigma \in \Omega \subset R^q$ is ***Lebesgue*** measurable. Ω is compact subset in R^q. $E(t,x,\sigma): R\times R^n \times \Omega \rightarrow R^{m\times m}$ is the uncertainty of input gain.

* This work was supported by the NSF of China under Grant 60274046.

$\Delta f(x)$ is nonlinear uncertainty. $I \in R^{m\times m}$ is unit matrix.

The uncertainty of system's input gain $E(t,x,\sigma)$ is piecewise continuous for variable t, and is continuous for x,σ. The uncertainty $\Delta f(x)$ is smooth for x. The additional disturbance $d(t)$ is piecewise continuous vector function. Under the above condition, we can know that the systems (1) have unique solution for any initial state and piecewise continuous control input $u(t)$.

We give the following assumptions for systems (1).

Assumption 1 The pair $(A(t),B(t))$ are uniformly completely controllable.

Assumption 2 There exist continuously nonnegative bounded functions $\eta(t),\rho(t),\lambda(t)$, $t\in R^+$ and nonnegative constants η^*,ρ^*,λ^*, such that

$$\|E(t,x,\sigma)\| \le \eta(t) \tag{2a}$$

$$\|\Delta f(x)\| \le \rho(t)\|x\| \tag{2b}$$

$$\|d(t)\| \le \lambda(t) \tag{2c}$$

where $\eta(t),\rho(t),\lambda(t)$; η^*,ρ^*,λ^* are unknown, and they satisfy $\eta(t)\le\eta^*<1$, $\rho(t)\le\rho^*$, $\lambda(t)\le\lambda^*$.

The condition $\eta^*<1$ in assumption 2 denotes that the uncertain gain in every input passage is less than normal input gain. Such assumption is reasonable.

Systems (1) satisfy assumption 1. So from the optimization control theory, the following ***Riccati*** matrix equation

$$-\dot{P}(t) = [A(t)+\alpha I]^{\mathrm{T}} P(t) + P(t)[A(t)+\alpha I] - \beta P(t)B(t)B^{\mathrm{T}}(t)P(t) + \gamma I \tag{3}$$

has positive definite solution $P(t)$. Where I is unit matrix with correspondent dimension, α,β,γ are designing parameters and more than zero.

We give the following design strategy of practically stable controller:

2.2 Design of practically stable adaptive robust controller

For the convenience of discussion, we denote

$$w_1(t) = \frac{1}{1-\eta^*}[\beta + \frac{1}{\gamma}\rho^2(t)] \tag{4a}$$

$$w_2(t) = \frac{1}{1-\eta^*}\lambda(t) \tag{4b}$$

where $w_1(t)$ can be taken as the uncertainty of the systems (1), $w_2(t)$ as the additional disturbance of systems (1). From the assumption 2, $w_1(t)$, $w_2(t)$ are also continuously bounded functions for $\forall t\in R^+$, and satisfy

$$w_1(t) \le w_1^* = \frac{1}{1-\eta^*}[\beta + \frac{1}{\gamma}(\rho^*)^2] \tag{5a}$$

$$w_2(t) \le w_2^* = \frac{1}{1-\eta^*}\lambda^* \tag{5b}$$

We design the following exponentially practically stable controller

$$u(t) = -\frac{1}{2}k(t)B^{\mathrm{T}}(t)P(t)x(t) \tag{6}$$

where the control gain function $k(t)$ is given by the following equality

$$k(t) = \hat{w}_1(t) + \tau^2\hat{w}_2^2(t) \tag{7}$$

The positive definite matrix $P(t)$ is given by ***Riccati*** equation (3), τ is positive designing parameter. $\hat{w}_1(t)$, $\hat{w}_2(t)$ are about the estimations of unknown parameters w_1^*, w_2^* respectively, and satisfy the adaptive law as

$$\dot{\hat{w}}_1 = -\delta_1\gamma_1\hat{w}_1 + \frac{1}{2}\gamma_1\left\|B^{\mathrm{T}}(t)P(t)x(t)\right\|^2 \tag{8a}$$

$$\dot{\hat{w}}_2 = -\delta_2\gamma_2\hat{w}_2 + \gamma_2\left\|B^{\mathrm{T}}(t)P(t)x(t)\right\| \tag{8b}$$

Then the closed loop systems consist of systems (1) and controller (6) are

$$\dot{x} = A(t)x(t) + B(t)(I+E(t))(-\frac{1}{2}k(t)B^{\mathrm{T}}(t)\times P(t)x(t)) + B(t)\Delta f(x) + B(t)d(t) \tag{9}$$

On the other hand, let

$$\psi_1(t) = \hat{w}_1(t) - w_1^* \tag{10a}$$

$$\psi_2(t) = \hat{w}_2(t) - w_2^* \tag{10b}$$

then we can change the equality (8) into the following error equation

$$\dot{\psi}_1 = -\delta_1\gamma_1\psi_1(t) + \frac{1}{2}\gamma_1\left\|B^{\mathrm{T}}(t)P(t)x(t)\right\|^2 - \delta_1\gamma_1 w_1^* \tag{11a}$$

$$\dot{\psi}_2 = -\delta_2\gamma_2\psi_2(t) + \gamma_2\left\|B^{\mathrm{T}}(t)P(t)x(t)\right\| - \delta_2\gamma_2 w_2^* \tag{11b}$$

Then the following theorem will give the result that the states of the closed-loop systems consist of systems (1) and error systems (11) are uniformly ultimately bounded:

Theorem 1 If systems (1) satisfies assumption 1-2, the solutions (x,ψ_1,ψ_2) $(t;t_0,x(t_0),\psi_1(t_0),\ \psi_2(t_0))$ of the closed-loop systems consist of systems (1), error systems (11) and controllers (6) are uniformly ultimately bounded. That is to say the states of systems (1) are practically stable under the effect of controllers (6) and adaptive laws (8), and the states of systems (1) are guaranteed to converge exponentially to the ball $B(0,r)$ which is centered by equilibrium point. The final bound r is determined by designing parameters and the parameters of systems (1) itself. Namely, the states of systems (1) can reach the prescribed region nearby the equilibrium point $x=0$ in limited time.

Proof: For studying the stability of the closed-loop systems consists of systems (9) and error systems (11), we construct the following ***Lyapunov*** function

$$V(t)=x^{\mathrm{T}}(t)P(t)x(t)+(1-\eta^*)\gamma_1^{-1}\psi_1^2+(1-\eta^*)\gamma_2^{-1}\psi_2^2 \qquad (12)$$

For the convenience of the proof, let

$$V_1(t)=x^{\mathrm{T}}(t)P(t)x(t)\,,\qquad V_2(t)=(1-\eta^*)\gamma_1^{-1}\psi_1^2\,,$$
$$V_3(t)=(1-\eta^*)\gamma_2^{-1}\psi_2^2$$

We calculate $\dot V_1(t)$, $\dot V_2(t)$, $\dot V_3(t)$ respectively.

$$\begin{aligned}\dot V_1(t)&=\dot x^{\mathrm{T}}(t)P(t)x(t)+x^{\mathrm{T}}(t)\dot P(t)x(t)+x^{\mathrm{T}}(t)P(t)\dot x(t)\\&=x^{\mathrm{T}}(t)[\dot P(t)+A^{\mathrm{T}}(t)P(t)+P(t)A(t)-k(t)\times\\&\quad PBB^{\mathrm{T}}P]x(t)-k(t)x^{\mathrm{T}}(t)PBEB^{\mathrm{T}}Px(t)+\\&\quad 2x^{\mathrm{T}}(t)PBd(t)+2x^{\mathrm{T}}(t)P(t)B(t)\Delta f(x)\end{aligned} \qquad (13)$$

Transform each item in above expression as following

$$-k(t)x^{\mathrm{T}}(t)P(t)B(t)E(t)B^{\mathrm{T}}(t)P(t)x(t)\le \eta^*k(t)\left\|B^{\mathrm{T}}(t)P(t)x(t)\right\|^2 \qquad (14a)$$

$$\begin{aligned}2x^{\mathrm{T}}(t)P(t)B(t)\Delta f(x)&\le 2\rho(t)\left\|B^{\mathrm{T}}(t)P(t)x(t)\right\|\left\|x(t)\right\|\\&\le\frac{1}{\gamma}\rho^2(t)\left\|B^{\mathrm{T}}(t)P(t)x(t)\right\|^2+\gamma\left\|x(t)\right\|^2\end{aligned} \qquad (14b)$$

$$2x^{\mathrm{T}}(t)P(t)B(t)d(t)\le 2\lambda(t)\left\|B^{\mathrm{T}}(t)P(t)x(t)\right\| \qquad (14c)$$

$$\begin{aligned}\dot V_2(t)&=2(1-\eta^*)\gamma_1^{-1}\psi_1\dot\psi_1=2(1-\eta^*)[-\delta_1\psi_1^2+\\&\quad\frac{1}{2}\psi_1\left\|B^{\mathrm{T}}(t)P(t)x(t)\right\|^2-\delta_1\psi_1w_1^*]\\&=-\delta_1\gamma_1V_2(t)+(1-\eta^*)\psi_1\left\|B^{\mathrm{T}}(t)P(t)x(t)\right\|^2-\\&\quad\delta_1(1-\eta^*)(\psi_1+w_1^*)^2+\delta_1(1-\eta^*)(w_1^*)^2\end{aligned} \qquad (15)$$

$$\begin{aligned}\dot V_3(t)&=2(1-\eta^*)\gamma_2^{-1}\psi_2\dot\psi_2=2(1-\eta^*)\times\\&\quad[-\delta_2\psi_2^2+\psi_2\left\|B^T(t)P(t)x(t)\right\|-\delta_2\psi_2w_2^*]\\&=-\delta_2\gamma_2V_3(t)+2(1-\eta^*)\psi_2\left\|B^T(t)P(t)x(t)\right\|-\\&\quad\delta_2(1-\eta^*)(\psi_2+w_2^*)^2+\delta_2(1-\eta^*)(w_2^*)^2\end{aligned} \qquad (16)$$

then from (3), (13), (14a)-(14c), we have the inequality

$$\begin{aligned}\dot V_1(t)&\le[-2\alpha x^{\mathrm{T}}(t)P(t)x(t)-\gamma\left\|x(t)\right\|^2+\\&\quad\beta\left\|B^{\mathrm{T}}(t)P(t)x(t)\right\|^2-(1-\eta^*)(\hat w_1(t)+\tau^2w_2^2(t))\times\\&\quad\left\|B^{\mathrm{T}}(t)P(t)x(t)\right\|^2+\frac{1}{\gamma}(\rho^*)^2\left\|B^{\mathrm{T}}(t)P(t)x(t)\right\|^2+\\&\quad\gamma\left\|x(t)\right\|^2+2\lambda^*\left\|B^{\mathrm{T}}(t)P(t)x(t)\right\|]\\&\le-2\alpha x^{\mathrm{T}}(t)P(t)x(t)-(1-\eta^*)[\hat w_1(t)+\\&\quad\tau^2w_2^2(t)]\left\|B^{\mathrm{T}}(t)P(t)x(t)\right\|^2+[\beta+\frac{1}{\gamma}(\rho^*)^2]\times\\&\quad\left\|B^{\mathrm{T}}(t)P(t)x(t)\right\|^2+2\lambda^*\left\|B^{\mathrm{T}}(t)P(t)x(t)\right\|\end{aligned} \qquad (17)$$

From (15), (17) and notice the value of w_1^* and (11a), we have

$$\begin{aligned}&\dot V_2(t)-[(1-\eta^*)\hat w_1-\beta-\frac{1}{\gamma}(\rho^*)^2]\left\|B^{\mathrm{T}}(t)P(t)x(t)\right\|^2\\&=-(1-\eta^*)(\hat w_1-w_1^*)\left\|B^{\mathrm{T}}(t)P(t)x(t)\right\|^2-\delta_1\gamma_1V_2(t)+\\&\quad(1-\eta^*)\psi_1\left\|B^{\mathrm{T}}(t)P(t)x(t)\right\|^2-\delta_1(1-\eta^*)\times\\&\quad(\psi_1+w_1^*)^2+\delta_1(1-\eta^*)(w_1^*)^2\\&\le-\delta_1\gamma_1V_2(t)+\delta_1(1-\eta^*)(w_1^*)^2\end{aligned} \qquad (18)$$

From (16), (17) and pay attention to the value of w_2^*, (11b), we have

$$\begin{aligned}&\dot V_3(t)+2\lambda^*\left\|B^{\mathrm{T}}(t)P(t)x(t)\right\|-(1-\eta^*)\tau^2\times\\&\quad w_2^2(t)\left\|B^{\mathrm{T}}(t)P(t)x(t)\right\|^2\\&=2\lambda^*\left\|B^{\mathrm{T}}(t)P(t)x(t)\right\|-(1-\eta^*)\tau^2w_2^2(t)\times\\&\quad\left\|B^{\mathrm{T}}(t)P(t)x(t)\right\|^2-\delta_2\gamma_2V_3(t)+2(1-\eta^*)\psi_2\times\\&\quad\left\|B^{\mathrm{T}}(t)P(t)x(t)\right\|-\delta_2(1-\eta^*)(\psi_2+w_2^*)^2+\delta_2(1-\eta^*)(w_2^*)^2\\&\le-(1-\eta^*)\tau^2w_2^2(t)\left\|B^{\mathrm{T}}(t)P(t)x(t)\right\|^2+2(1-\eta^*)\times\\&\quad(\psi_2+w_2^*)\left\|B^{\mathrm{T}}(t)P(t)x(t)\right\|-\delta_2\gamma_2V_3(t)+\delta_2(1-\eta^*)(w_2^*)^2\\&=-(1-\eta^*)[\tau\hat w_2(t)\left\|B^{\mathrm{T}}(t)P(t)x(t)\right\|-\frac{1}{\tau}]^2+\frac{(1-\eta^*)}{\tau^2}-\\&\quad\delta_2\gamma_2V_3(t)+\delta_2(1-\eta^*)(w_2^*)^2\\&\le-\delta_2\gamma_2V_3(t)+\delta_2(1-\eta^*)(w_2^*)^2+\frac{(1-\eta^*)}{\tau^2}\end{aligned} \qquad (19)$$

Therefore, from (17)-(19) it can be gotten

$$\dot V(t)=[\dot V_1(t)+\dot V_2(t)+\dot V_3(t)]\le-2\alpha V_1(t)-\delta_1\gamma_1V_2-\delta_2\gamma_2V_3$$

$$+\frac{(1-\eta^*)}{\tau^2}+\delta_1(1-\eta^*)(w_1^*)^2+\delta_2(1-\eta^*)(w_2^*)^2] \quad (20)$$

Let $\delta=\min\{\delta_1\gamma_1,\delta_2\gamma_2\}$ (21a)

$$\tilde{\mu}=\min\{2\alpha,\delta\} \quad (21b)$$

$$\varepsilon=\frac{(1-\eta^*)}{\tau^2}+(1-\eta^*)[\delta_1(w_1^*)^2+\delta_2(w_2^*)^2] \quad (21c)$$

then $\dot{V}(t)\le-\tilde{\mu}V(t)+\varepsilon$ (22)

From (12), (22), it can be obtained that ***Lyapunov*** function $V(x(t),\psi_1(t),\psi_2(t))$ decrease monotonously along the states of closed-loop systems until the compact set

$$\Omega_f=\{(x(t),\psi_1(t),\psi_2(t))\mid V(t)\le V_f\} \quad (23)$$

where $V_f=\tilde{\mu}^{-1}\varepsilon$ (the solution of (22) when the right part equal to zero) (24)

From (22)-(24), we can get the solutions $(x,\psi_1,\psi_2)(t;t_0,x(t_0),\psi_1(t_0),\psi_2(t_0))$ of the closed-loop systems consists of systems (1), error systems (11) and controller (6) are uniformly ultimately bounded, that is to say, closed-loop systems are practically stable.

According to (22)~(24), the states of systems (1) converge exponentially to the ball $B(0,r)$ which is centered by equilibrium point, where the degree $\tilde{\mu}$ is determined by (21b). The final bound r is decided by (22). Therefore, the states of systems (1) satisfy $\|x(t)\|^2\le\mu^{-1}\tilde{\mu}^{-1}\varepsilon$, where μ is the minimum eigenvalue of positive definite matrix P.

Remark 1 From (21a) and (21b), we know the degree $\tilde{\mu}$ of exponential convergence can be adjusted by parameters α, β, δ_i, γ_i $(i=1,2)$, according to the requirement of the designer. From (21c) and $\|x(t)\|^2\le\mu^{-1}\tilde{\mu}^{-1}\varepsilon$, the upper bound of the stable solution $x(t)$ can be adjusted by the value of the parameters δ_i. Then the upper bound satisfies the requirement of designer. Therefore, the controllers (6) and adaptive laws (8) can guarantee the states of systems (1) to reach the prescribed region in limited time.

3 Simulation

Consider the following time-vary linear dynamic systems

$$\dot{x}(t)=A(t)x(t)+B(t)[I+E(t,x,\sigma)]u(t)+B(t)\Delta f(x)+B(t)d(t) \quad (25)$$

where $A=\begin{pmatrix}0 & 1/(t+1)\\ 1/(2t+1) & 1\end{pmatrix}$, $B=\begin{pmatrix}0\\1\end{pmatrix}$, $I=1$, $E=-0.3\sin(10t)$, $\Delta f(x)=(\sin(10t)\ \ \cos(10t))x=x_1\sin(10t)+x_2\cos(10t)$, $d(t)=0.8\sin(5t)$. Assume $\alpha=0.2$, $\beta=1$, $\gamma=1$. the initial conditions are $x(0)=(0.8,-0.9)^T$, $w_i(0)=0$, adjustive parameters are: $\gamma_1=0.3$, $\gamma_2=0.2$, $\delta_1=0.1$, $\delta_2=0.3$, $\tau=10$. Figures 1-2 show the results of the simulation

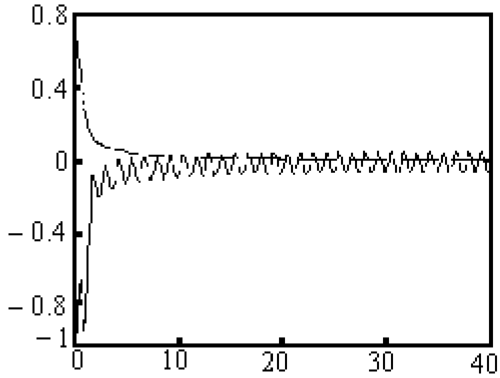

Figure 1: States of subsystems 1 "x_1 _._", x_2 ___"

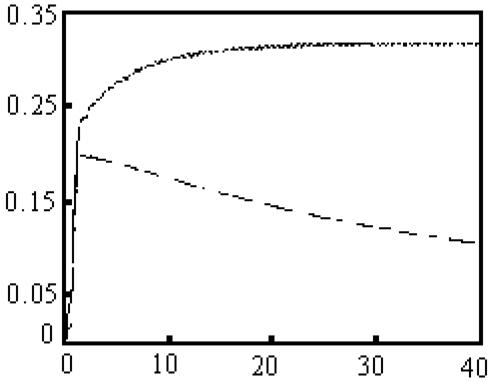

Figure 2: Adaptive variable "w_1 _._", w_2 ___"

From the above figures, we can see states of systems getting a fluctuation in the first several seconds, because there are additional disturbances. The controllers can not get information of feedback information promptly. When the controllers get the information of feedback, the states of systems decline immediately to the prescribed region under the effect of the designing controllers. There are no fluctuation in the state curves, if we get rid of the additional disturbance. Because the controllers that we constructed have strong robustness, they can still make the states of the

Proceedings of the Second International Conference on Machine Learning and Cybernetics, Xi'an, Nov.2003, pp.980-984

systems converge to the final attractor ($B(0,r)$) quickly. We can change the size of the attractor by adjusting the parameters properly to satisfy the requirment of the designer.

4 Conclusion

A kind of time-varying uncertain system with additional disturbance is studied in this paper. The designing scheme of nonlinear and adaptive controllers that guarantee the states of the systems uniformly ultimately bounded is given. From the result of the research, the designing controllers have stronger robustness comparing to the previous controllers that were designed based on the known bounds of the uncertainties. The result is closer to the practice.

References

[1] Gutman S. Uncertain dynamical systems – A Lyapunov min-max approach, IEEE Transactions Automatic Control, 24(3): 437-443, 1979

[2] Corless M J and Leitmann G. Continuous state feedback guaranteeing uniform ultimate boundedness for uncertain dynamic systems, IEEE Transactions Automatic Control, 26(5): 1139-1144, 1981

[3] Wu H and Mizukami K. Exponential stability of a class of nonlinear dynamical systems with uncertainties, Systems & Control Letters, 21: 307-313, 1993

[4] Choi C.H and Kim H.S. Adaptive regulation for a class of uncertain systems with partial knowledge of uncertainty bounds, IEEE Transactions Automatic Control, 38(8): 1246-1250, 1993

[5] Wu H and Shigemaru S. Linear adaptive robust controllers for a class of uncertain dynamical systems with unknown bounds of uncertainties, Proc.14th IFAC World Congress, Beijing, Vol.G, pp419-424, 1999